Автономная навигация мобильных роботов: биомиметика и математика

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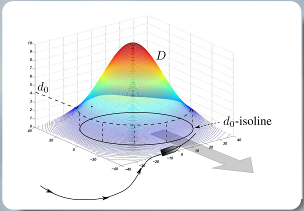
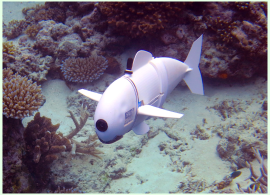
Department of
Theoretical Cybernetics

ITMO University
Faculty of
Control Systems and Robotics

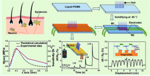


Biomimetics in robotics

Hardware design



Tactile sensor



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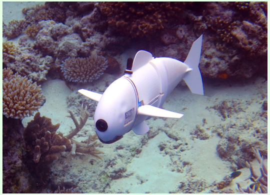

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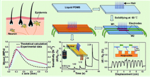

Algorithms and software

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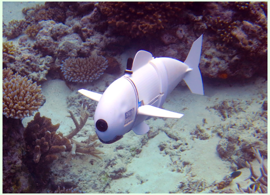

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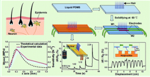

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Robots' Navigation

The aim of navigation is searching a (optimal or suboptimal) path from the start point to the destination point with obstacle avoidance competence.

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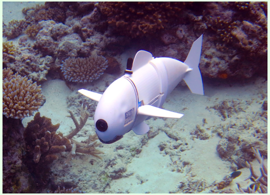

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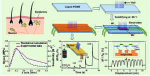

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Self-localisation + Path planning
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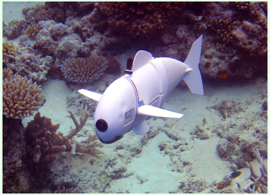

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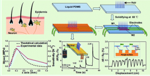

Algorithms and software

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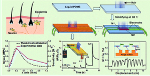

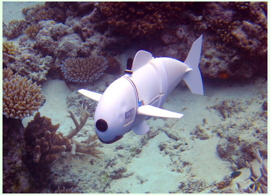

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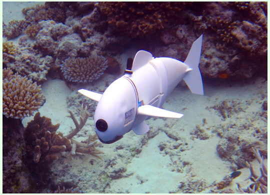

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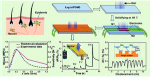

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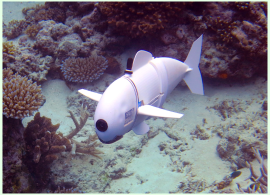

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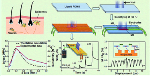

Algorithms and software

- Navigation is the process of determining and maintaining a course or trajectory to a destination location
- Environment: Structured (known, predictable) ↔ Unstructured (unknown, unpredictable)
- Planning horizon: Global ↔ Local ↔ Reactive
- Kinematic control

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Biomimetics in robotics

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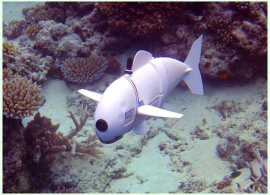

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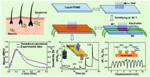

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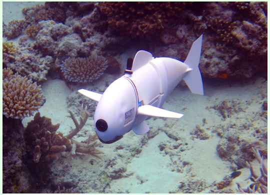

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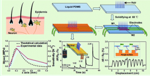

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- Kinematic control
- Guarantees of goal attainment
- under conditions that are close to those necessary for the mission feasibility

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The aim of navigation is searching a (optimal or suboptimal) path from the start point to the destination point with obstacle avoidance competence.

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Examples with popular insects

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Honey bees navigation

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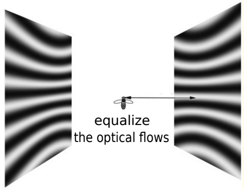

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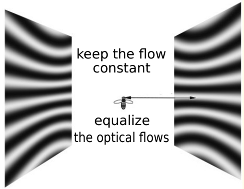

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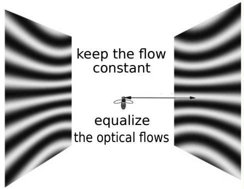

Ants navigation

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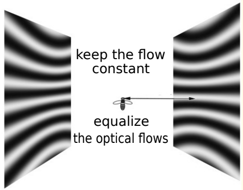

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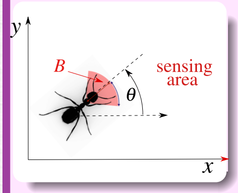

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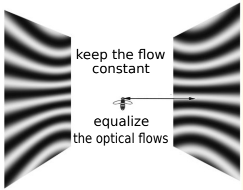

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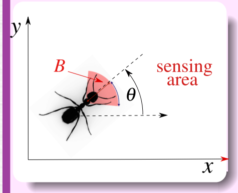


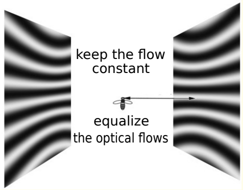
$$\dot{\boldsymbol{r}} = \vec{v}, \quad \frac{d\vec{v}}{dt} = \frac{1}{\tau}\left[\vec{v} - DV(\boldsymbol{r})\right],$$

$P(\boldsymbol{r})$– pheromone concentration at location $\boldsymbol{r}$

$$DV = \lambda \frac{1}{\int_B P(\boldsymbol{r})\, d\boldsymbol{r}} \int_B \boldsymbol{r} P(\boldsymbol{r})\, d\boldsymbol{r}$$

Examples with popular insects

Honey bees navigation



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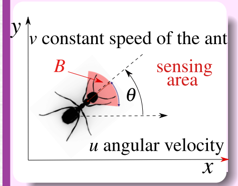

$$\dot{x} = v\cos\theta, \quad \dot{\theta} = u,$$
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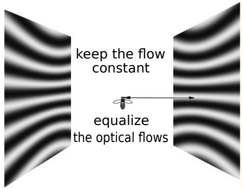

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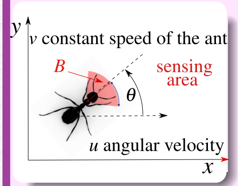

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$R(\theta)$rotation through angle θ

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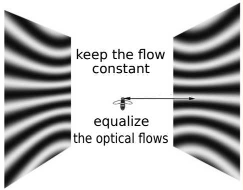

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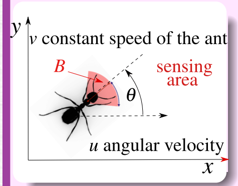

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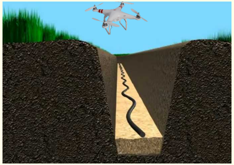
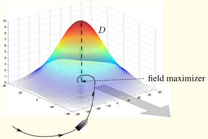
What is the practical aspect of this misery about ants





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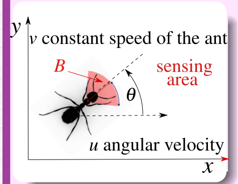

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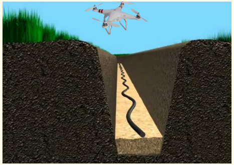
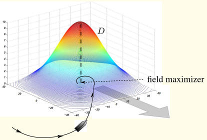
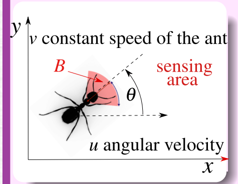




Does the ant succeed?

Ants navigation



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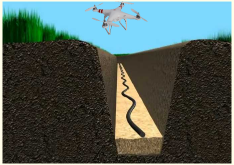

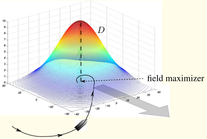

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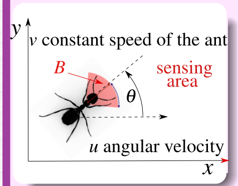

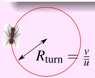

minimal turning radius

$$\begin{aligned} \dot{x} &= v\cos\theta, & \dot{\theta} &= u, \\ \dot{y} &= v\sin\theta, & |u| &\le \overline{u}, \end{aligned} \quad ,$$

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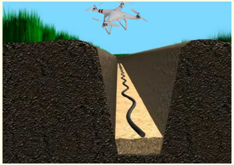

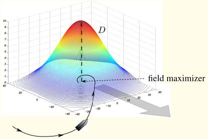

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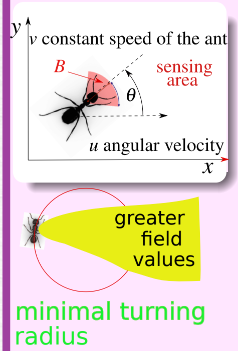

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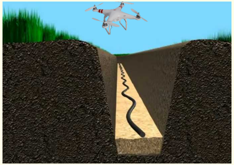

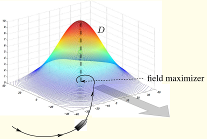

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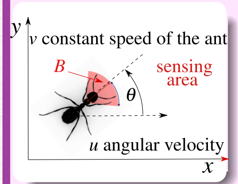

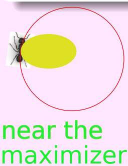

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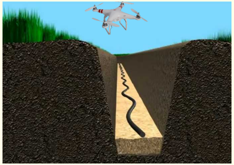
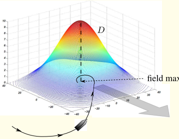
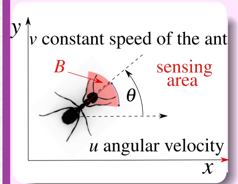




Does the ant succeed?

- The targeted vicinity of the maximizer $V_{\max}$ is so wide that the ant is able to remain there

Ants navigation



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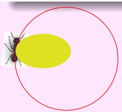

near the maximizer

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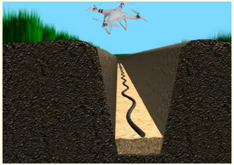

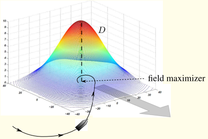

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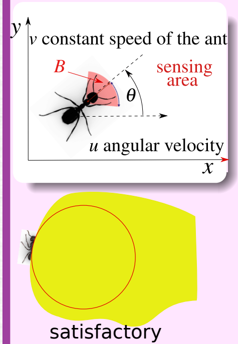

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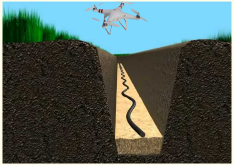

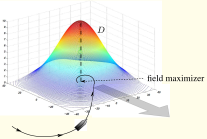

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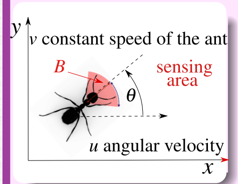

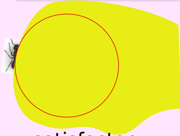

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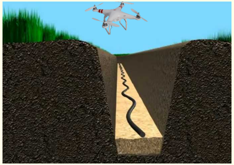

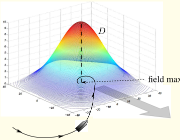

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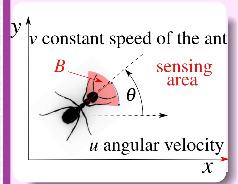

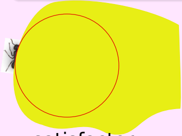

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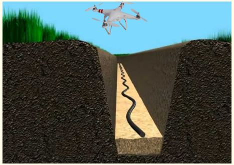

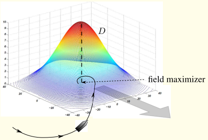

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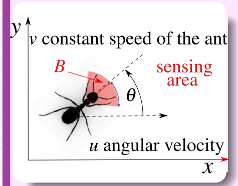
Ants navigation



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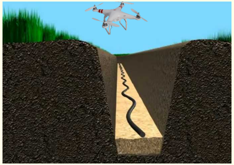

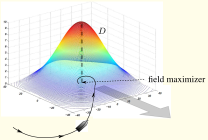

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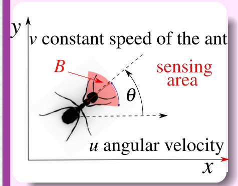

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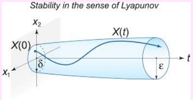

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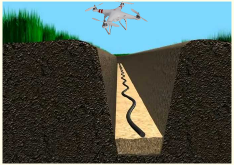

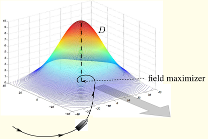

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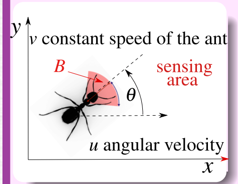

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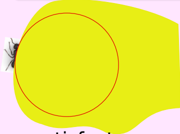

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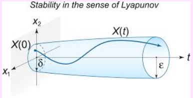

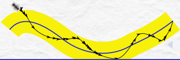

Inspiration point: peregrine falcon and equiangular navigation

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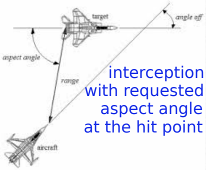

interception
with requested
aspect angle
at the hit point

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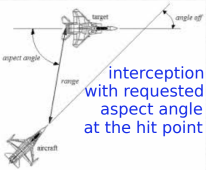

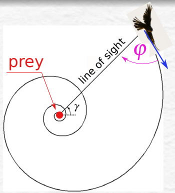

$\varphi = \text{const}$

Logarithmic spiral

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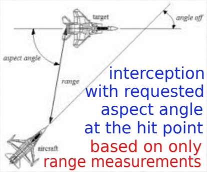

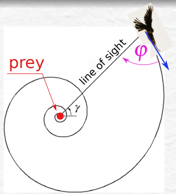

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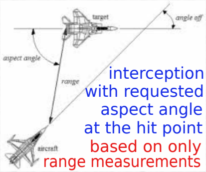

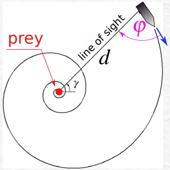

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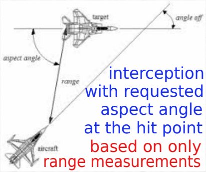

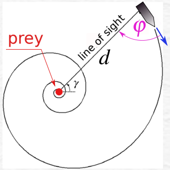

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$\dot{d} = -v \cos\varphi$, where v is the robots speed
$\dot{d} = -\nu = \text{const} \Rightarrow$ motion over a logarithmic spiral

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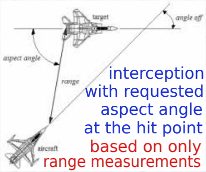

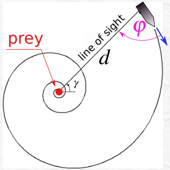

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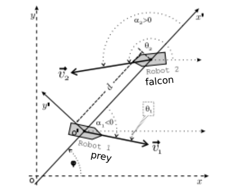
Dubins vehicle

$$\begin{array}{l} \dot x_i = v_i \cos\theta_i \\ \dot y_i = v_i \sin\theta_i \end{array}, \quad \begin{array}{l} \dot\theta_i = u_i \in [-\overline{u}_i, \overline{u}_i] \\ v_i \in [0, \overline{v}_1],\ v_2 = \text{const} \end{array}$$



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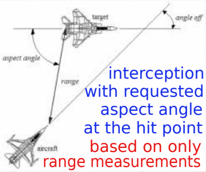

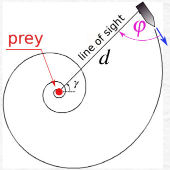

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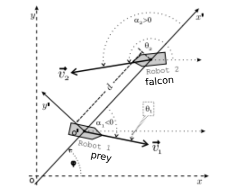

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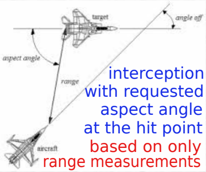

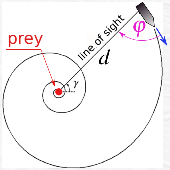

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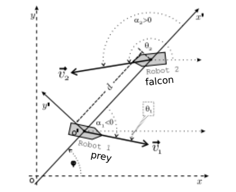

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Basic control paradigm

$$u_2 = \overline{u}_2 \mathbf{sgn}\left[\dot{d} - \nu\right]$$

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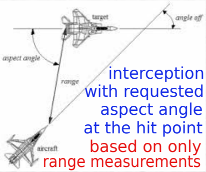

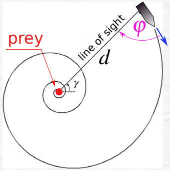

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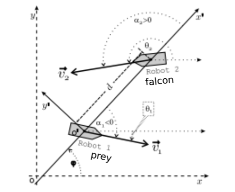

Basic control paradigm

$$u_2 = \overline{u}_2 \mathbf{sgn}\left[\dot{d} - \nu\right]$$

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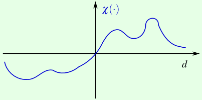
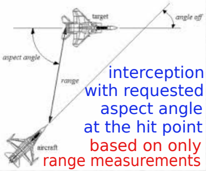
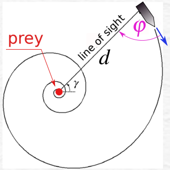


Inspiration point: peregrine falcon and equiangular navigation



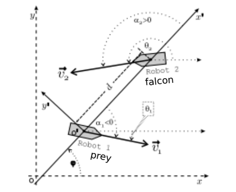


$\varphi =$ const
Logarithmic spiral

$\dot{d} = -v\cos\varphi$, where v is the robots speed
$\dot{d} = -\nu =$ const $\Rightarrow$ motion over a logarithmic spiral

Dubins vehicle

$$\begin{matrix} \dot{x}_i = v_i \cos\theta_i \\ \dot{y}_i = v_i \sin\theta_i \end{matrix}, \quad \begin{matrix} \dot{\theta}_i = u_i \in [-\overline{u}_i, \overline{u}_i] \\ v_i \in [0, \overline{v}_1], v_2 = \text{const} \end{matrix}$$



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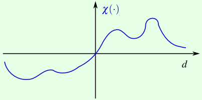
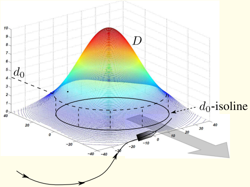


Extensions

- d is not necessarily the distance to a single pointwise target

Many targets, extended targets, cumulative strength of a signal, value of a scalar field



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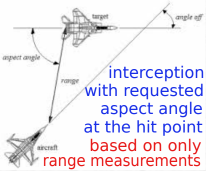
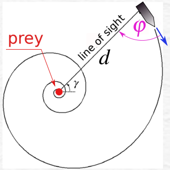


interception with requested aspect angle at the hit point based on only range measurements



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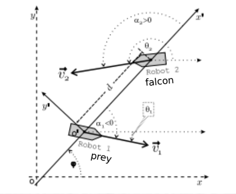

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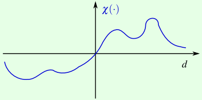

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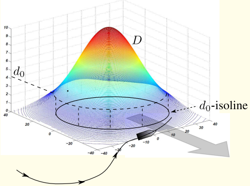
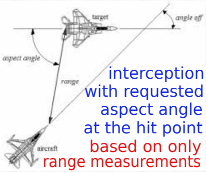


- Not necessarily a Dubins-car like robot
extensions on 3 dimensions

Inspiration point: peregrine falcon and equiangular navigation



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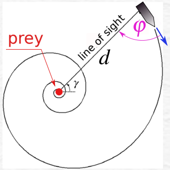

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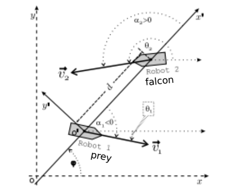

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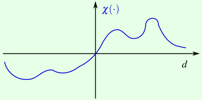

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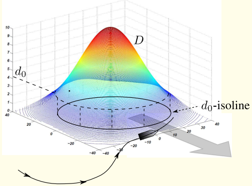

- Not necessarily a Dubins-car like robot
  extensions on 3 dimensions
- To whom much is given, from him it will be asked:
  Convergence under conditions nearly necessarily to the mission feasibility

Discontinuous control laws and sliding mode regimes

Toy example

$\dot{x} = u \in \mathbb{R}$, objective: $x \to 0$

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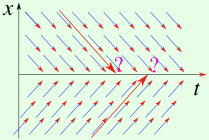

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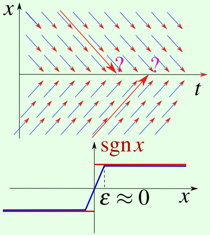

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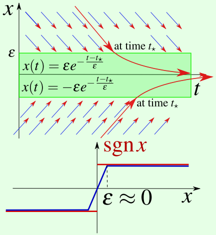

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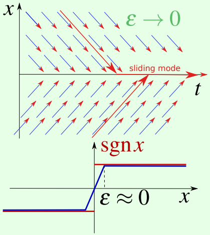

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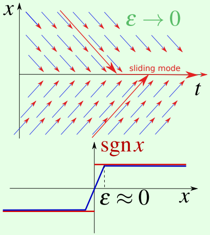

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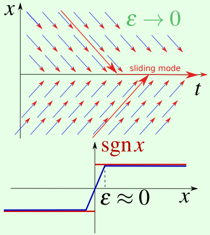

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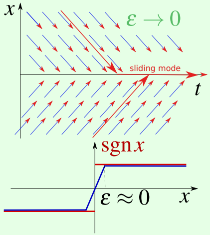

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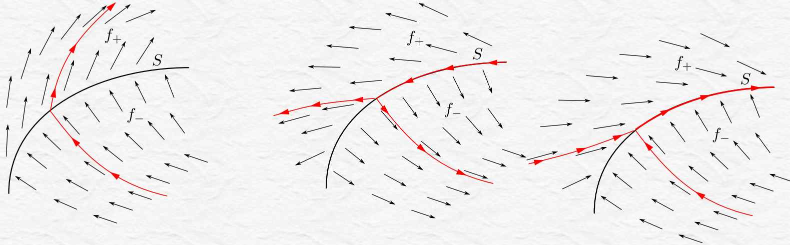

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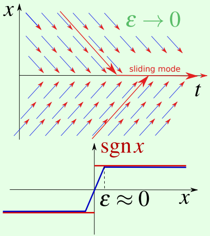

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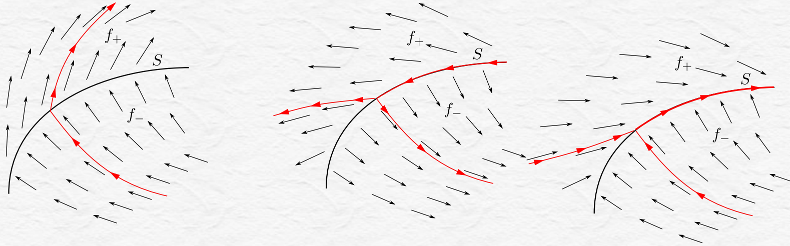

$$\dot{x} \begin{cases} \text{is tangent to the discontinuity surface} \\ \text{lies on the straight line segment with the ends } f_-(x) \text{ and } f_+(x) \end{cases}$$

Discontinuous control laws and sliding mode regimes

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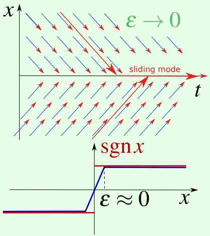

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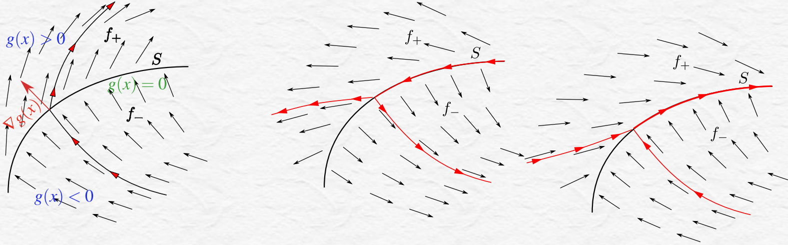

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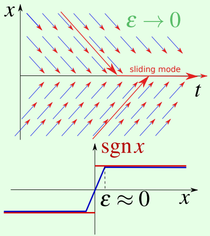

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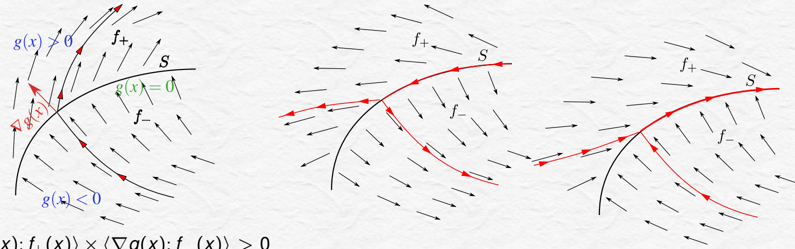

$$\langle \nabla g(x); f_+(x) \rangle \times \langle \nabla g(x); f_-(x) \rangle > 0$$

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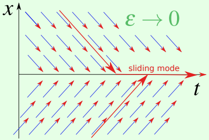

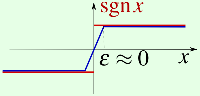

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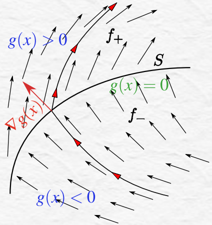

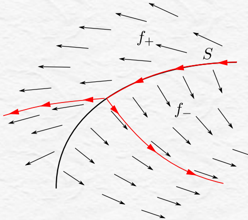

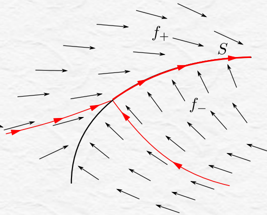

$\langle \nabla g(x); f_+(x) \rangle \times \langle \nabla g(x); f_-(x) \rangle > 0$

$\langle \nabla g(y); f(y) \rangle = \frac{dg[x(t)]}{dt} |_{t=t_*}$
if $\dot{x} = f(x), x(t_*) = y$

Discontinuous control laws and sliding mode regimes

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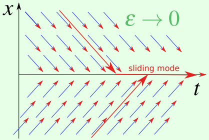

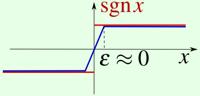

$$\dot{x} = \begin{cases} -1 & \text{if } x \geq \varepsilon \\ -\frac{x}{\varepsilon} & \text{if } -\varepsilon < x < \varepsilon \\ 1 & \text{if } x \leq -\varepsilon \end{cases}$$

General case

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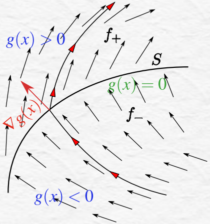
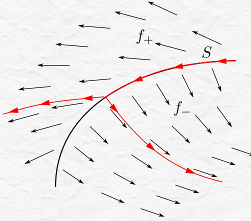
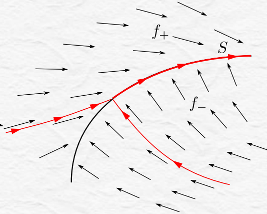






$$\lim_{y \to x, g(y) > 0} \frac{dg}{dt} \times \lim_{y \to x, g(y) < 0} \frac{dg}{dt} > 0$$

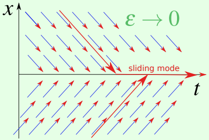
$$\langle \nabla g(y); f(y) \rangle = \frac{dg[x(t)]}{dt} \Big|_{t=t_*}$$
if $\dot{x} = f(x), x(t_*) = y$

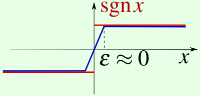
Discontinuous control laws and sliding mode regimes

Toy example

$$\dot{x} = u \in \mathbb{R}, \text{ objective: } x \to 0$$

$$\text{control law: } u = -\mathbf{sgn}\, x$$





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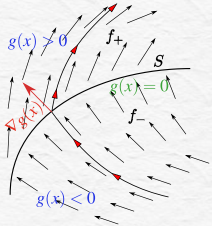

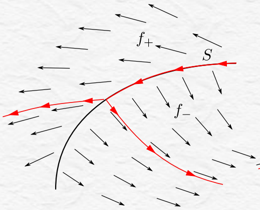

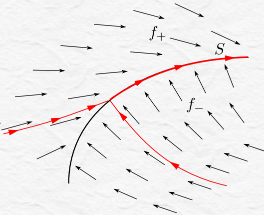

$$\lim_{y \to x, g(y)>0} \frac{dg}{dt} \times \lim_{y \to x, g(y)<0} \frac{dg}{dt} > 0$$

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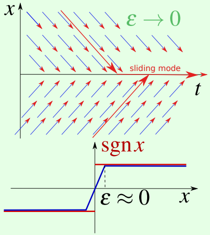

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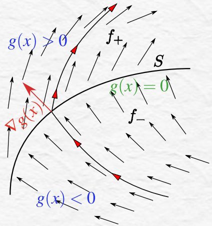

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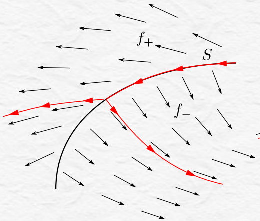

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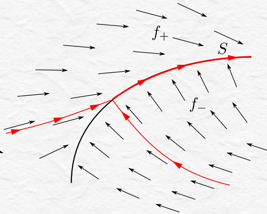
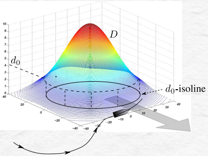


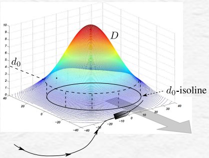
$$\lim_{\substack{y \to x \\ g(y) > 0}} \frac{dg}{dt} < 0, \; \lim_{\substack{y \to x \\ g(y) < 0}} \frac{dg}{dt} > 0$$



$D(x, y) \in \mathbb{R}$ unknown unimodal scalar field

Robot measures only the field value d at its current location

Robot Navigation for Monitoring Environmental Boundaries without Field Gradient Estimation



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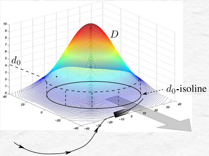

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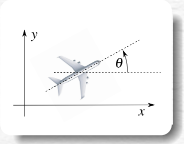

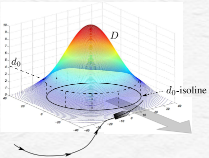

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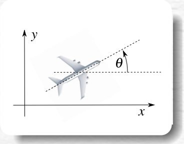

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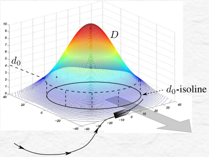

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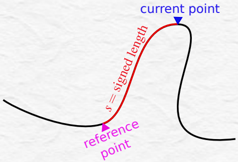

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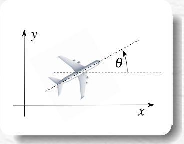

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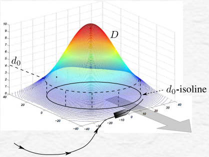

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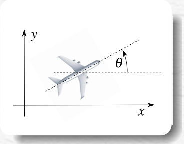

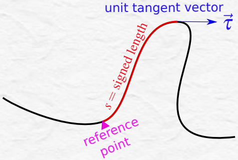

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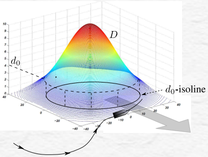

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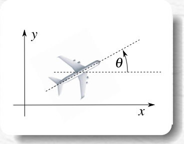
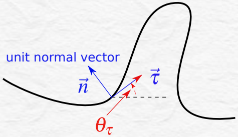
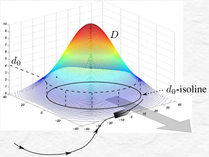


Frenet-Serrat frame





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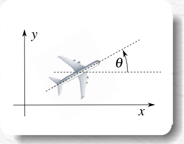

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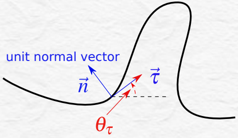

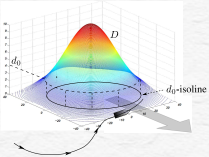

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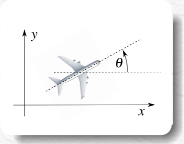

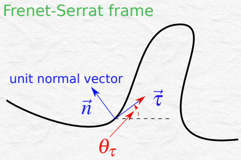

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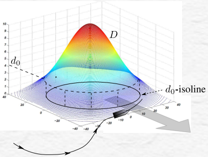
Paths trackable by the robot



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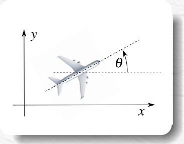

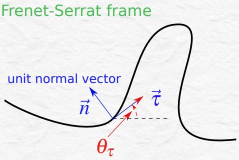

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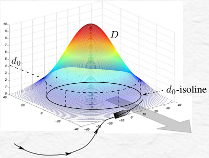
$$r(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = p[s(t)]$$



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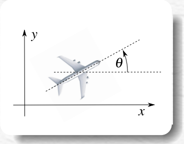

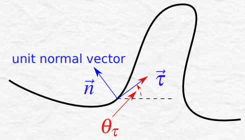

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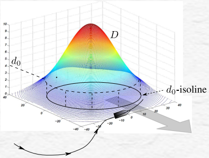

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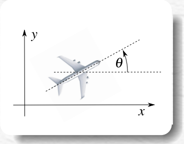
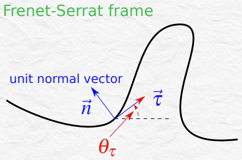
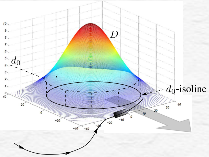




Paths trackable by the robot

$$r(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = p[s(t)] \Leftrightarrow r(0) = p[s(0)] \text{ and } \dot{r}(t) = \frac{d}{dt}p[s(t)] \forall t \geq 0 \quad \dot{s}(t) \equiv \pm v$$



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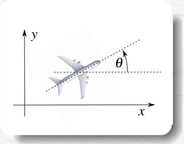

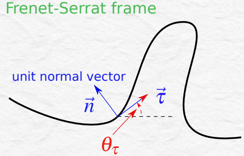

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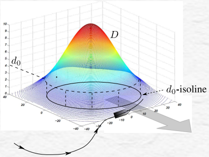

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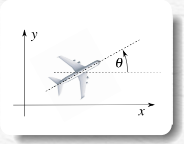

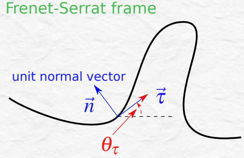

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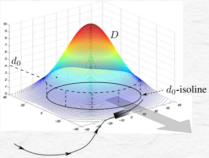

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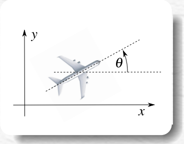

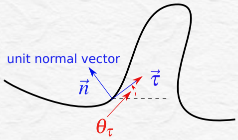

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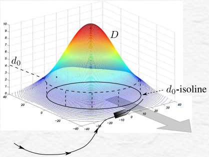

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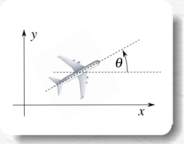

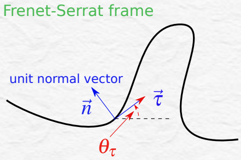

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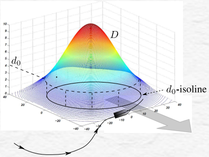

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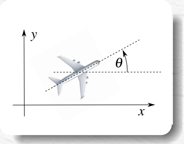

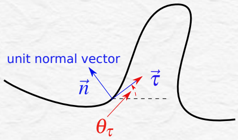

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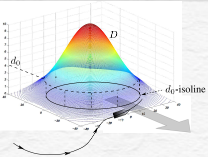

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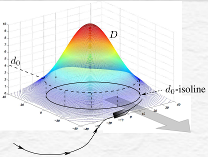

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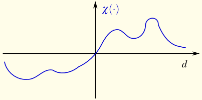
$$\omega = -\overline{\omega}\,\mathbf{sgn}\left[\dot{d} + \mu\chi(d - d_0)\right]$$



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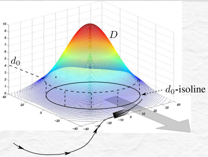

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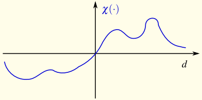

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State variables and discontinuity surface

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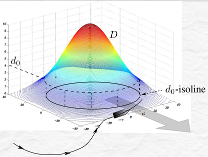
$$g = (\boldsymbol{r}, \theta) = D(\boldsymbol{r})$$

discontinuity is described by

$$g(\boldsymbol{r}, \theta) := \underbrace{v \left\langle \nabla D(\boldsymbol{r}); \vec{e}(\theta) \right\rangle}_{\dot{d}} + \mu\chi\left[d(x, y) - d_0\right] = 0$$



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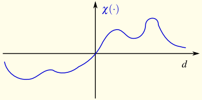

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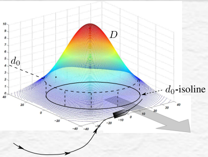
Захват робота неизвестным полем



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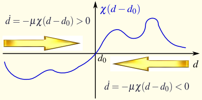

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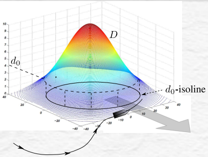

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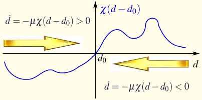

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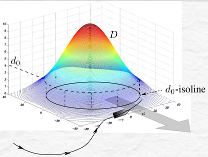

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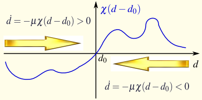

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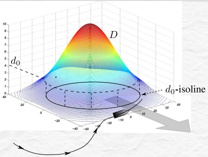

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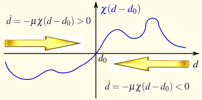

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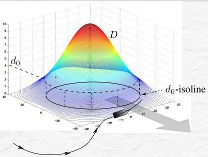

$D(x, y) \in \mathbb{R}$ unknown unimodal scalar field

Primary constraints

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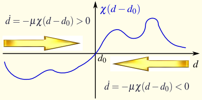

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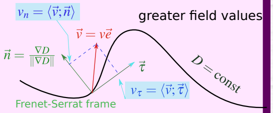


Robot measures only the field value d at its current location

State variables and discontinuity surface

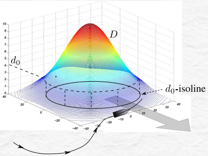
state $(\boldsymbol{r}, \theta)$

$g = (\boldsymbol{r}, \theta) = D(\boldsymbol{r})$

discontinuity is described by

$$g(\boldsymbol{r}, \theta) := \underbrace{v\left\langle \nabla D(\boldsymbol{r}); \vec{e}(\theta)\right\rangle}_{\dot{d}} + \mu\chi\left[d(x, y) - d_0\right] = 0$$

Захват робота неизвестным полем



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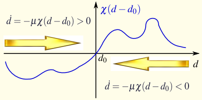

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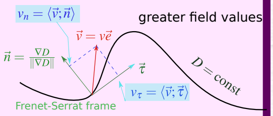

$$|v_n| = \left|\left\langle v\vec{e}; \frac{\nabla D}{\|\nabla D\|}\right\rangle\right| = \frac{v\left|\left\langle \nabla D; \vec{e}\right\rangle\right|}{\|\nabla D\|} = \frac{|\dot{d}|}{\|\nabla D\|} < v$$

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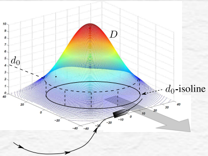

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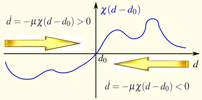

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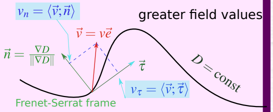

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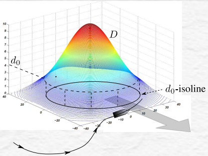

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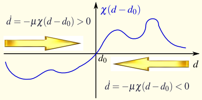

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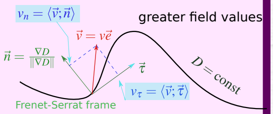

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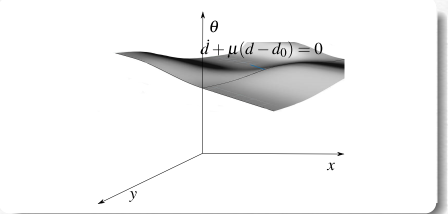

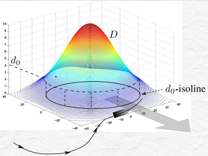

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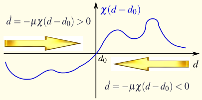

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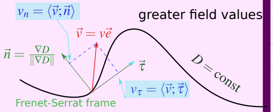

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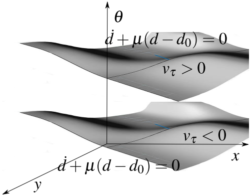

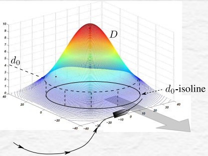

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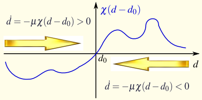

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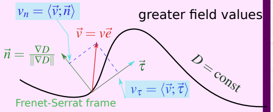

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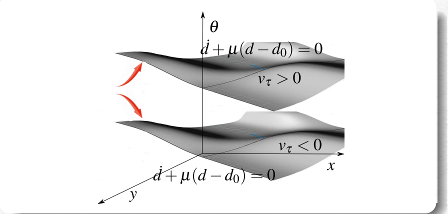

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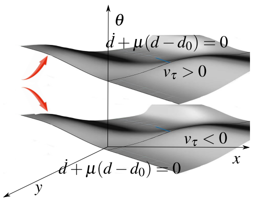

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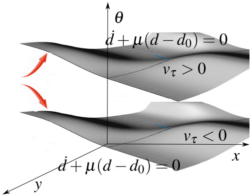

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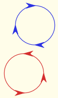

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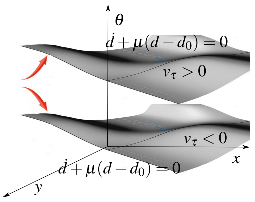

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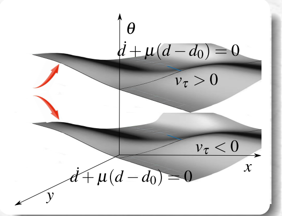

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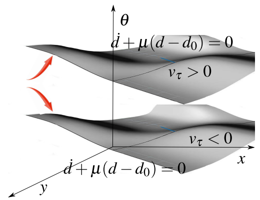

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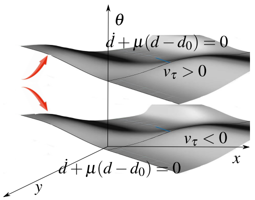
$$\lim_{(\boldsymbol{r},\theta)\to(\boldsymbol{r}_*,\theta_*),g>0} \frac{dg}{dt} > 0, \quad \lim_{(\boldsymbol{r},\theta)\to(\boldsymbol{r}_*,\theta_*),g<0} \frac{dg}{dt} < 0$$

State variables and discontinuity surface

state $(\boldsymbol{r}, \theta)$

discontinuity is described by

$$g(\boldsymbol{r},\theta) := \underbrace{v\left\langle \nabla D(\boldsymbol{r}); \vec{e}(\theta)\right\rangle}_{\dot{d}} + \mu\chi\left[d(x,y) - d_0\right] = 0$$



$$\frac{d\vec{e}}{d\theta} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} =: \vec{e}_\perp$$

$$\frac{dg}{dt} = v^2\left\langle D''\vec{e}; \vec{e}\right\rangle + v\omega\left\langle \nabla D(\boldsymbol{r}); \vec{e}_\perp\right\rangle + \mu\frac{d\chi}{dt}$$

Robot's model

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Control law

$$\omega = -\overline{\omega}\,\mathbf{sgn}\left[\dot{d} + \mu\chi(d - d_0)\right] = -\overline{\omega}\,\mathbf{sgn}\,g$$

$$g > 0 \Rightarrow \omega = -\overline{\omega}$$

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Attracting and sliding

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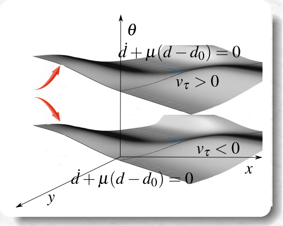

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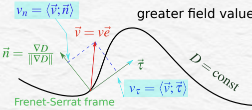

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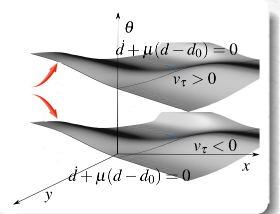

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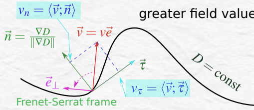

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Attracting and sliding

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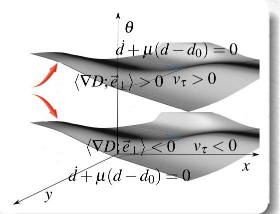

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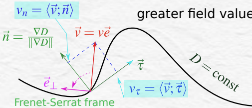

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$$\lim_{(\boldsymbol{r},\theta)\to(\boldsymbol{r}_*,\theta_*),g>0} \frac{dg}{dt} < 0, \quad \lim_{(\boldsymbol{r},\theta)\to(\boldsymbol{r}_*,\theta_*),g<0} \frac{dg}{dt} > 0$$

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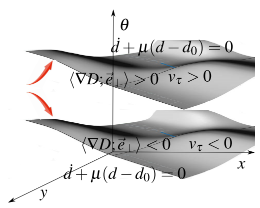

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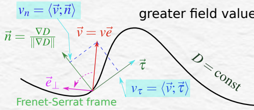

$$\frac{dg}{dt} = v^2\left\langle D''\vec{e}; \vec{e}\right\rangle + v\omega\left\langle \nabla D(\boldsymbol{r}); \vec{e}_\perp\right\rangle + \mu\frac{d\chi}{dt}$$

$$\overset{\mu\approx 0}{\approx} v^2\|\nabla D\|\left[\frac{\langle D''\vec{e};\vec{e}\rangle}{\|\nabla D\|} + \frac{\omega}{v}\left\langle \frac{\nabla D(\boldsymbol{r})}{\|\nabla D\|}; \vec{e}_\perp\right\rangle\right]$$

Захват робота неизвестным полем

Robot's model

$$\dot{\boldsymbol{r}} = v\vec{e}(\theta), \quad \dot{\theta} = \omega$$

$$\boldsymbol{r} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \vec{e} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

Control law

$$\omega = -\overline{\omega}\,\mathbf{sgn}\left[\dot{d} + \mu\chi(d - d_0)\right] = -\overline{\omega}\,\mathbf{sgn}\, g$$

$$g > 0 \Rightarrow \omega = -\overline{\omega}$$

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Attracting and sliding

$$\lim_{(\boldsymbol{r},\theta)\to(\boldsymbol{r}_*,\theta_*),g>0} \frac{dg}{dt} < 0, \quad \lim_{(\boldsymbol{r},\theta)\to(\boldsymbol{r}_*,\theta_*),g<0} \frac{dg}{dt} > 0$$

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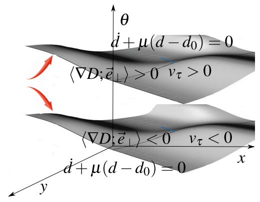
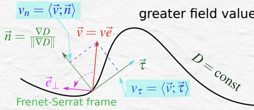


$$\vec{e}_\perp \quad \frac{d\vec{e}}{d\theta} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} =:$$



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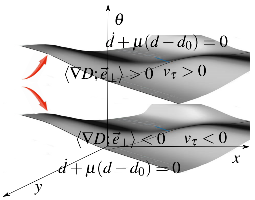

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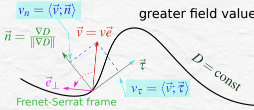

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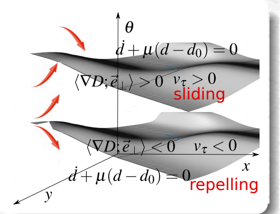

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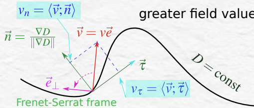

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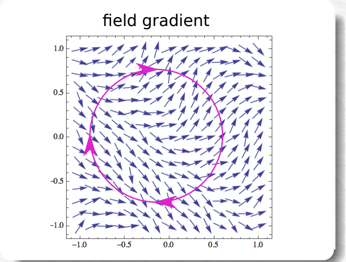
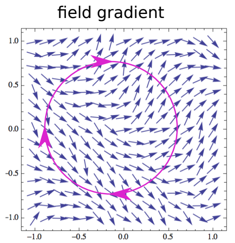
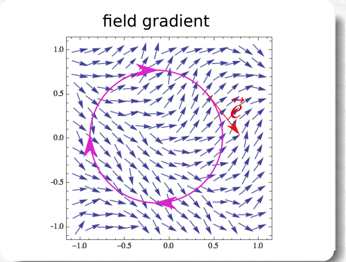


$$\dot{d} + \mu\chi(d - d_0) = ?$$



$$v \langle \nabla D; \vec{e} \rangle + \mu \chi (d - d_0) = ?$$



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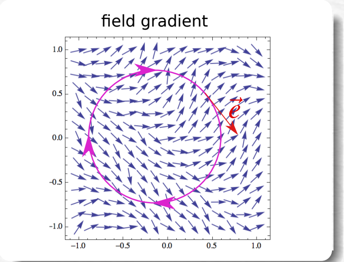

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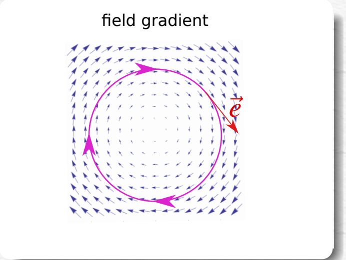
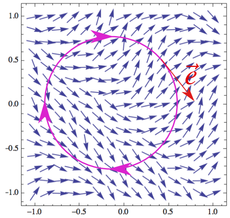


field gradient



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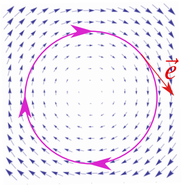
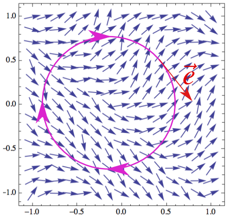


In any simply connected domain

If $\nabla D \neq 0$ and is continuous, then there exists a continuous function $\alpha(\boldsymbol{r})$ (polar angle) such that $\nabla D(\boldsymbol{r}) = \|\nabla D(\boldsymbol{r})\| \begin{bmatrix} \cos \alpha(\boldsymbol{r}) \\ \sin \alpha(\boldsymbol{r}) \end{bmatrix}$

field gradient



$$v \langle \nabla D; \vec{e} \rangle + \mu \chi(d - d_0) = ?$$

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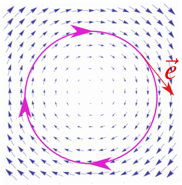

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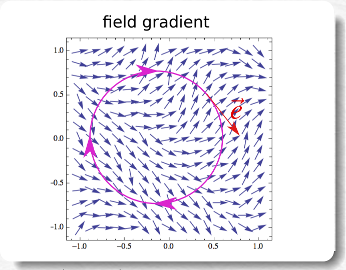
angle between $\vec{e}$ and ∇D = polar angle of $\vec{e}$ − polar angle of ∇D



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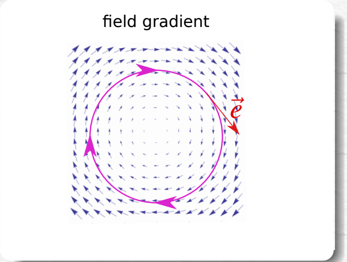

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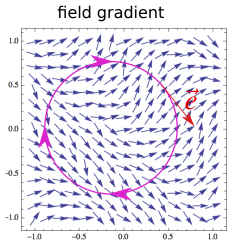
angle between $\vec{e}$ and ∇D = polar angle of $\vec{e} - \alpha(\boldsymbol{r})$



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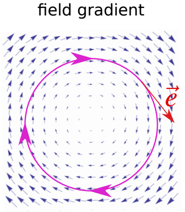

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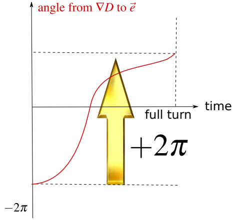
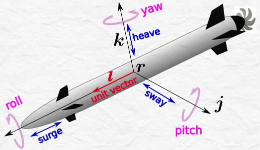
angle between $\vec{e}$ and ∇D = polar angle of $\vec{e} - \alpha(\boldsymbol{r})$





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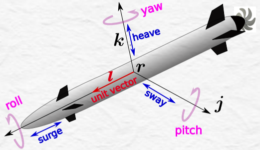
Here $\langle \cdot ; \cdot \rangle$ is the inner product,
q is the pitch and r is the yaw rate



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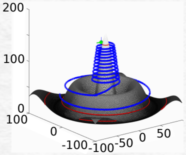
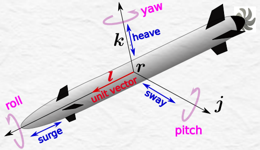
- Find, arrive at, and then repeatedly sweep the isosurface within a given range of altitudes $[h_-, h_+]$





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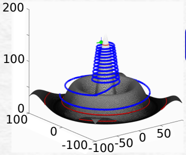
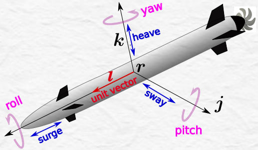


Necessary conditions for trackability of the isosurface

are given in terms of its front speed and acceleration, principal curvature, rate of rotation, density of isolines and the rates of its change



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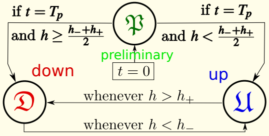
Control law: switching



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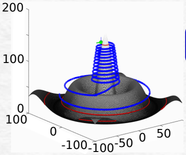

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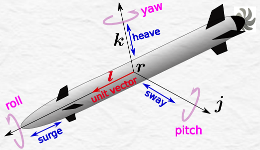

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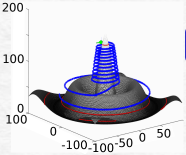

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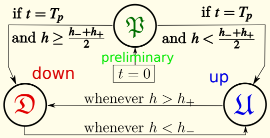


Control law: continuous regulation

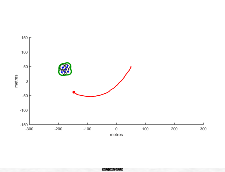
$$v_{\text{vert}} := 0 \text{ in } \mathfrak{P}, \quad v_{\text{vert}} := v_h \text{ in } \mathfrak{U}, \quad v_{\text{vert}} := -v_h \text{ in } \mathfrak{D}$$

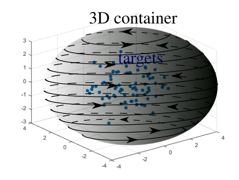
$$\boldsymbol{u} = -\overline{u}_h \cdot \mathbf{sgn}\left[\dot{h} - v_{\text{vert}}\right] \boldsymbol{h}_{\text{y-p}} + \overline{u}_d \cdot \mathbf{sgn}\left[\dot{d} + \mu\chi(d - d_0)\right] \boldsymbol{h}_{\text{y-p}} \times \vec{\imath}$$

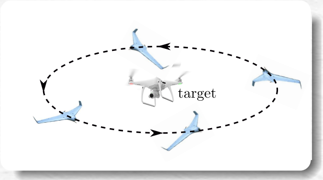
where $v_h, \overline{u}_d, \overline{u}_h, \mu$ are controller parameters, $\boldsymbol{h}_{\text{y-p}}$ is the projection of the vertical vector onto the yaw-pitch plane of the robot normalized to the unit length

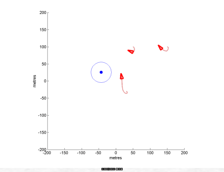
Zoo of some elaborated applications

- Searching, circumnavigating, and following both single and multiple unpredictably maneuvering targets by a single robot and robotic team
  - Distributed control, effective self-distribution
  - Kinematics (nonholonomy, underactuation) and dynamics constraints

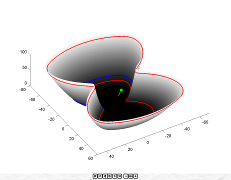






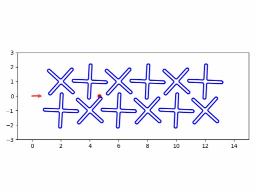


- Tight surface scan by a mobile robot



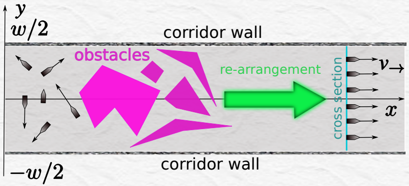
Zoo of some elaborated applications

- Environmental extremum seeking in 2D and 3D by single and multiple robots, both steady and dynamic fields, kinematics (e.g., nonholonomy and underactuation) and dynamics constraints
- Tracking environmental level sets in 2D and 3D by single and multiple robots, maze-like environments
- Border patrolling and obstacle avoidance; moving and deforming obstacles



- 3D navigation in tunnel-like environments,
- Decentralized sweep boundary coverage
- Distributed self-deployment of robotic networks; barrier and sweep coverage
- Autonomous unmanned helicopter in unknown urban environments
- Multiple wheeled robots in unknown cluttered environments/ Unmanned agricultural tractor / Motorized mobile hospital bed

Sweep coverage of corridor environments with an obstacle course



- Cannot distinguish among the peers
- No communication facilities
- Cannot play distinct roles
- Unaware of the team's size and the corridor width
- The obstacles are unknown
- Has access to the corridor direction and relative positions of the objects within a finite distance if the view of the object is unobstructed by an obstacle.

Control law inspired by collective behavior of animal spieces

$$v_i^x := v_\rightarrow + \frac{1}{|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i}\Upsilon[x_j - x_i],$$

$$v_i^y := \Xi[d_{i,\delta}^+] - \Xi[d_{i,\delta}^-]$$

$$+\max_{j\in\mathcal{N}_i} h[y_j - y_i]w_j^- + \min_{j\in\mathcal{N}_i} h[y_j - y_i]w_j^+,$$

Finish

The end

Dear colleagues, thank you for your kind attention

Please enjoy the rest of this meeting.